

$B^\pm \rightarrow DK^\pm$ with direct CP violation in charm

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We investigate the implications of direct CP violation (CPV) on the determination of the unitarity triangle angle γ from $B \rightarrow DK$ decays. We show that γ can still be extracted even with the inclusion of direct CPV in charm if (i) at least one of the D decays has negligible CP violation; and (ii) data from charm factory at threshold are used. If approximate expressions without including direct CP violation in charm are used, this can result in a shift in γ that is $\mathcal{O}(r_D/r_B)$. It is modest for $B \rightarrow DK$ but can be $\mathcal{O}(1)$ for $B \rightarrow D\pi$. We illustrate the size of the shift on an example of Gronau-London-Wyler method.

I. INTRODUCTION

Both LHCb [1] and CDF [2] have measured nonzero CP violation in the charm sector giving a combined value for the difference of CP asymmetries

$$\Delta a_{CP} = a_{CP}(K^- K^+) - a_{CP}(\pi^- \pi^+) = (-6.45 \pm 1.80) \times 10^{-3}. \quad (1)$$

Here $a_f \equiv a_{CP}(f)$ is the time integrated CP asymmetry for D decaying to a CP eigenstate f

$$a_f \equiv \frac{\Gamma(D^0 \rightarrow f) - \Gamma(\bar{D}^0 \rightarrow f)}{\Gamma(D^0 \rightarrow f) + \Gamma(\bar{D}^0 \rightarrow f)}. \quad (2)$$

In general a_f is a sum of three contributions

$$a_f = a_f^{\text{dir}} + a_f^m + a_f^i, \quad (3)$$

where a_f^{dir} is CP violation in the decay (or direct CP violation), a_f^m CP violation from mixing and a_f^i CP violation from interference of decay and mixing [4]. Mixing effects are universal, independent of the final state, and are furthermore suppressed by the $D^0 - \bar{D}^0$ mixing parameters $x, y \sim \mathcal{O}(10^{-2})$ and are in general negligible (the exact size does depend on the time interval the experiments integrate over, however). For instance, LHCb quotes [1]

$$a_{K^+ K^-} - a_{\pi^+ \pi^-} \approx a_K^{\text{dir}} - a_\pi^{\text{dir}} + (0.10 \pm 0.01)a_{\text{ind}}, \quad (4)$$

so that $a_{\text{ind}} = a^m + a^i$ can be safely neglected.

In the present manuscript we are interested in the effect that nonzero a_f^{dir} can have on the methods for determining the CKM unitarity triangle angle, γ , [5]

$$\gamma = \frac{V_{ud}V_{cb}V_{ub}^*V_{cd}^*}{V_{ud}^*V_{cb}^*V_{ub}V_{cd}} \approx (66 \pm 12)^\circ, \quad (5)$$

from $B \rightarrow D^{(*)}K^{(*)}$ decays. These methods rely on the interference of $B \rightarrow D^0 K \rightarrow (f)_D K$ and $B \rightarrow \bar{D}^0 K \rightarrow$

$(f)_D K$ amplitudes, where f is a common final state to both D^0 and \bar{D}^0 decays [6–9] (see also reviews in [10, 11]). Due to the interference one can probe the relative phase of the two amplitudes, which is related to γ . In the originally proposed methods an important assumption was that there is no CP violation in D decays¹. Below we relax this assumption and show that the effect of CP violation in charm can be included. For a completely general treatment additional information is needed. If this information is not included we show how big of an error is introduced when assuming no CPV in charm decays.

II. GENERAL CONSEQUENCES OF CP VIOLATION IN CHARM

We first focus on singly Cabibbo suppressed decays. The D^0 (\bar{D}^0) decay amplitudes A_f (\bar{A}_f) to CP eigenstate f can in general be written as

$$A_f \equiv A(D^0 \rightarrow f) = A_f^T e^{i\phi_f^T} [1 + r_f e^{i(\delta_f + \phi_f)}], \quad (6)$$

$$\bar{A}_f \equiv A(\bar{D}^0 \rightarrow f) = A_f^T e^{-i\phi_f^T} [1 + r_f e^{i(\delta_f - \phi_f)}], \quad (7)$$

where $A_f^T e^{\pm i\phi_f^T}$ is the dominant singly-Cabibbo suppressed tree amplitude. In general it can have a weak phase ϕ_f^T , however, in the SM this is zero. Since we are primarily interested in the extraction of γ assuming the SM, we will set $\phi_f^T = 0$ in the following. The ratio r_f denotes the relative magnitude of the subleading amplitude due to penguin diagrams, while δ_f and ϕ_f are the strong and weak phase differences, respectively. Direct CP asymmetry is then

$$a_f^{\text{dir}} = -\frac{2r_f \sin \delta_f \sin \phi_f}{1 + 2r_f \cos \delta_f \cos \phi_f + r_f^2} \approx -2r_f \sin \delta_f \sin \phi_f, \quad (8)$$

where the last expression is valid for $r_f \ll 1$ which is a good approximation in D decays.

In order to have CP violation in the SM all three generations need to participate. This means that in D decays the

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¹ In [6–9] D^0 - \bar{D}^0 mixing was also neglected. The effects of $x_D \neq 0$ in the measurement of γ are discussed in [12–15]. D^0 - \bar{D}^0 mixing will be also neglected throughout the present manuscript.

CP violation is suppressed by the CKM elements between the third and the first two generations, giving a naive estimate $r_f \sim \mathcal{O}([V_{cb}V_{ub}/V_{cs}V_{us}]\alpha_s/\pi) \sim 10^{-4}$. The penguin contraction matrix elements seem to be enhanced in the SM, giving an estimate $r_f \sim 10^{-3}$ and the observed size of Δa_{CP} [16, 17] (see also [18–20]). The enhanced value or r_f could also be due to NP [21]. In this paper we are interested solely on the effect of the observed Δa_{CP} on the extraction of γ from $B \rightarrow DK$. We can set aside the origin of Δa_{CP} , as our conclusion will not change, as long as the tree contributions are SM-like and do not carry weak phase.

The $B^- \rightarrow DK^-$ amplitudes are

$$A(B^- \rightarrow D^0 K^-) \equiv A_B, \quad (9)$$

$$A(B^- \rightarrow \bar{D}^0 K^-) \equiv A_B r_B e^{i(\delta_B - \gamma)}, \quad (10)$$

where $r_B = 0.099 \pm 0.008$, while $\delta_B = (110 \pm 15)^\circ$ is the strong phase [5].² The amplitude for $B^- \rightarrow f_D K^-$ decay is then

$$A(B^- \rightarrow f_D K^-) = A_B A_f^T [1 + r_B e^{i(\delta_B - \gamma)} + r_f e^{i(\delta_f + \phi_f)} + r_f r_B e^{i(\delta_B - \gamma + \delta_f - \phi_f)}]. \quad (11)$$

The amplitude for the CP conjugated process is obtained by flipping the signs of all the weak phases, $\gamma \rightarrow -\gamma, \phi_f \rightarrow -\phi_f$,

$$A(B^+ \rightarrow f_D K^+) = A_B A_f^T [1 + r_B e^{i(\delta_B + \gamma)} + r_f e^{i(\delta_f - \phi_f)} + r_f r_B e^{i(\delta_B + \gamma + \delta_f + \phi_f)}]. \quad (12)$$

We first discuss the errors introduced by neglecting the effect of CPV in charm. If $r_f = 0$, we have

$$A(B^- \rightarrow f_D K^-) = A_B A_f^T [1 + r_B e^{i(\delta_B - \gamma)}], \quad (13)$$

$$A(B^+ \rightarrow f_D K^+) = A_B A_f^T [1 + r_B e^{i(\delta_B + \gamma)}]. \quad (14)$$

Now for $r_f \neq 0$. Keeping just linear terms in r_f, r_B we have

$$A(B^- \rightarrow f_D K^-) = A_B A_f^T [1 + r_B e^{i(\delta_B - \gamma)} + r_f e^{i(\delta_f - \phi_f)}], \quad (15)$$

$$A(B^+ \rightarrow f_D K^+) = A_B A_f^T [1 + r_B e^{i(\delta_B + \gamma)} + r_f e^{i(\delta_f + \phi_f)}]. \quad (16)$$

which can be re-written as

$$A(B^- \rightarrow f_D K^-) = A_B A_f^T [1 + r_B^- e^{i(\delta_B' - \gamma - \delta\gamma)}], \quad (17)$$

$$A(B^+ \rightarrow f_D K^+) = A_B A_f^T [1 + r_B^+ e^{i(\delta_B' + \gamma + \delta\gamma)}]. \quad (18)$$

Where, at $\mathcal{O}(r_f)$, $\delta\gamma = (r_f/r_B) \cos(\delta_f - \delta_B) \sin(\phi_f - \gamma)$. So the shift in γ is $\mathcal{O}(r_f/r_B)$. This is the case since sensitivity to γ arises at $\mathcal{O}(r_B)$ therefore all corrections need to be compared to the size of the smaller amplitude in (13), (14).

For each final state f there is also a shift in the δ_B strong phase, $\delta_B' - \delta_B = (r_f/r_B) \sin(\delta_f - \delta_B) \cos(\phi_f - \gamma)$ as well as in the ratio of the two CKM amplitudes, which now exhibits direct CP violation $r_B^\mp - r_B = (r_f/r_B) \cos[\delta_f - \delta_B \mp (\phi_f - \gamma)]$. In order to extract the correct value of γ all of the above corrections need to be kept. Note that the CP asymmetry at $\mathcal{O}(r_f, r_B)$ takes a simple form

$$A_{CP}(B \rightarrow f_D K) = 2r_B \sin \delta_B \sin \gamma + a_f^{\text{dir}}, \quad (19)$$

so that the expressions can be easily corrected at this order by measured quantities. The CP averaged branching ratio on the other hand is

$$Br(B \rightarrow f_D K) = A_B^2 A_f^{T2} [1 + 2r_B \sin \delta_B \sin \gamma + a_f^{\text{dir}} \cot(\delta_f)], \quad (20)$$

so that the correction requires the knowledge of strong phase δ_f . This phase can be measured using the charm-factories as we show below. For completeness we also give in appendix A the unexpanded expressions for branching ratios and CP asymmetries.

Next we show that in principle one can still get γ exactly even if there is CPV in charm decays. That is we can still over-constrain the system and obtain all the unknowns directly from experiments. To show this we first count the number of parameters. Let n_B be the number of different B decays, for instance $B \rightarrow DK$, $B \rightarrow DK^*$, $B \rightarrow D\pi$, and let n_{CA} and n_{SCS} be the number of Cabibbo allowed and singly Cabibbo suppressed D decays, respectively. Only SCS D decays are assumed to have nonzero CP violation, with both the branching ratios and CP asymmetries measured. Also, ϕ_f^T is assumed to be zero, which is the case in the SM. Each SCS D decay depends on four parameters, A_f^T , r_f , δ_f , and ϕ_f which can be reduced using the measured branching ratio, $Br(D \rightarrow f)$, and direct CP asymmetry a_{CP}^{dir} . We thus have two new independent parameters per SCS D decay. For each B decay we have three new unknowns, A_B , r_B , δ_B , and a common unknown, γ . Therefore the total number of unknowns is

$$\text{Unkn} : 3n_B + 1 + 2n_{SCS}. \quad (21)$$

The total number of observables is the number of different $B \rightarrow f_D K$ branching ratios and CP asymmetries, which is

$$\text{Obs} : 2n_B(n_{CA} + n_{SCS}). \quad (22)$$

Since this grows quadratically, while the number of unknowns grows linearly, for n 's high enough the system can be over-constrained. For instance, for $n_B = 2$, $n_{CA} = 1$, $n_{SCS} = 2$ we have 12 measurables and 11 unknowns. So at least in principle the system is solvable. We reiterate that we assumed that there is no CPV in CA decays, and as we show next this is crucial.

² For simplicity of notation we focus on $B \rightarrow DK$ decay, but the results apply also to $B \rightarrow D^* K$ and $B \rightarrow DK^*$ decays. The ratio of amplitudes and the strong phase difference are then $r_B(D^* K) = 0.121_{-0.019}^{+0.018}$, $\delta_B(D^* K) = (-55_{-16}^{+14})^\circ$ and $r_B(DK^*) = 0.118 \pm 0.045$, $\delta_B(DK^*) = (117_{-42}^{+30})^\circ$ [5].

In order to obtain general understanding of when γ can be extracted let us define a CP violating phase

$$\alpha_f \equiv \arg(A_f/\bar{A}_f). \quad (23)$$

To first order in r_f , $\alpha_f = 2r_f \cos \delta_f \sin \phi_f$. For $B \rightarrow DK$ decays one then has

$$|A(B^\mp \rightarrow f_D K^\mp)|^2 = |A_B|^2 [|A_f|^2 + r_B^2 |\bar{A}_f|^2 + 2r_B |A_f| |\bar{A}_f| \cos(\delta_B \mp \gamma \mp \alpha_f)]. \quad (24)$$

From this we see that there is an overall shift symmetry $\gamma \rightarrow \gamma + \phi$, $\alpha_f \rightarrow \alpha_f - \phi$. This shift symmetry is not broken by measuring a_f^{dir} . Since $\alpha_f = a_f^{\text{dir}} \cot \delta_f$, a change in α_f can always be compensated by adjusting the unknown strong phase difference δ_f .

The shift symmetry has an important implication for γ extraction. In fact the angle γ cannot be extracted from $B \rightarrow DK$ data alone, unless we can disentangle the γ and α_f dependence, e.g. by assuming that at least one of the D decays has no CP violation, $\alpha_f = 0$. This is the case despite the fact that we showed above that the total number of observables can be made bigger than the number of unknowns. The assumption $\alpha_f = 0$ is valid in the SM for the Cabibbo allowed and Cabibbo doubly suppressed D decays. However, the shift symmetry is so far broken only from the interference terms in the B decay and would thus require more statistics.

One can make further progress by using the fact that $r_f \ll 1$ in any reasonable explanation of Δa_{CP} . Expanding to first order in r_f one then arrives at Eqs. (19), (20). A nice observation was made by LHCb collaboration in [22], where they noted that $a_f^{\text{dir}} \cot(\delta_f)$ appears in the same form for $Br(B \rightarrow f_D K)$ and $Br(B \rightarrow f_D \pi)$ (but with $r_B(B \rightarrow f_D \pi) \ll 1$). In the ratio $Br(B \rightarrow f_D K)/Br(B \rightarrow f_D \pi)$ thus the term $a_f^{\text{dir}} \cot(\delta_f)$ cancels. The inclusion of a_{CP}^{dir} then gives only a marginal shift in γ in the analysis of [22].

One can also use $B \rightarrow f_D K$ decays separately if the interference terms are measured using charm factories running at $\Psi(3770)$. Then the relative phases $\alpha_f - \alpha_{f'}$ can be measured from entangled decays. For instance, for $\Psi(3770) \rightarrow D\bar{D} \rightarrow f_D f'_D$ decays, where both $D \rightarrow f$ and $D \rightarrow f'$ are SCS we have

$$\Gamma(\Psi(3770) \rightarrow f_D f'_D) \propto \alpha_f - \alpha_{f'}, \quad (25)$$

where we expanded to leading power in $r_f, r_{f'}$. If one of the two final states is Cabibbo allowed (we take it to be f'), then

$$\Gamma(\Psi(3770) \rightarrow f_D f'_D) \propto 1 - 2 \frac{\bar{A}_{f'}}{A_{f'}} \cos(\alpha_f). \quad (26)$$

The ratio of DCS and CA amplitudes is small, $\bar{A}_{f'}/A_{f'} \sim \lambda^2 \sim 0.05$, while in addition the sensitivity to α_f is only at quadratic order, $\cos \alpha_f = 1 - \alpha_f^2/2 + \dots$. Therefore the $\Psi(3770) \rightarrow D\bar{D}$ entangled decay with a CA decay on one side and SCS decay on the other side is not optimal for breaking the shift symmetry degeneracy.

A better strategy is to use Dalitz plot decays, where both CA and SCS decays interfere, for instance in $D^0 \rightarrow K_S \pi^+ \pi^-$. In this Dalitz plot the $D^0 \rightarrow K_S \rho^0$ and $D^0 \rightarrow K^{*-} \pi^+$ amplitudes interfere. We have

$$\Gamma(D^0 \rightarrow K_S \pi^+ \pi^-) \propto |A_{K_S \rho^0} BW_{\rho^0}(m_+^2, m_-^2) + A_{K^{*-} \pi^+} BW_{K^{*-}}(m_+^2, m_-^2) + \dots|^2, \quad (27)$$

where $BW_{\rho^0, K^{*-}}$ are the Breit-Wigner functions, $m_\pm^2 = (p_{K_S} + p_{\pi^\pm})^2$, and the ellipses denote the other resonances in the Dalitz plot. For \bar{D}^0 decay we have similarly

$$\Gamma(\bar{D}^0 \rightarrow K_S \pi^+ \pi^-) \propto |\bar{A}_{K_S \rho^0} BW_{\rho^0}(m_+^2, m_-^2) + \bar{A}_{K^{*+} \pi^-} BW_{K^{*+}}(m_+^2, m_-^2) + \dots|^2. \quad (28)$$

Because of the known strong phase variation in the Breit-Wigner functions one can extract the phase differences $\arg(A_{K_S \rho^0}/A_{K^{*-} \pi^+})$ and $\arg(\bar{A}_{K_S \rho^0}/\bar{A}_{K^{*+} \pi^-})$ by measuring the corresponding interference regions in the D^0 and \bar{D}^0 decays. Since CA decays $D^0 \rightarrow K^{*-} \pi^+$ and $\bar{D}^0 \rightarrow K^{*+} \pi^-$ do not carry a weak phase (as is the case in the SM) we also have $\arg(\bar{A}_{K^{*+} \pi^-}/A_{K^{*-} \pi^+}) = 0$. As a result one can extract the relative phase $\alpha_{K_S \rho^0}$ between the $D^0 \rightarrow K_S \rho^0$ and $\bar{D}^0 \rightarrow K_S \rho^0$ decay amplitudes. Once $\alpha_{K_S \rho^0}$ is measured one can use charm-factory to measure the CPV phases α_f for the remaining SCS decays such as $D \rightarrow K^+ K^-, \pi^+ \pi^-, \dots$, from $\Psi(3770) \rightarrow f_D f'_D$ decays with SCS D decays on both sides.

III. GRONAU-LONDON-WYLER DETERMINATION OF γ

We now show the consequences of direct CPV in charm on determining γ using an explicit example – the Gronau-London-Wyler (GLW) method [6, 7]. We shorten the notation following [23] and define

$$z_1 \equiv \bar{A}_f A_B, \quad z'_1 \equiv A_f A_B, \quad (29)$$

$$z_2 \equiv A_f A_{B'} r_B e^{i(\delta_B + \gamma)}, \quad z'_2 \equiv \bar{A}_f A_{B'} r_B e^{i(\delta_B - \gamma)}, \quad (30)$$

so that

$$z \equiv A(B^+ \rightarrow f_D K^+) = z_1 + z_2, \quad (31)$$

$$z' \equiv A(B^- \rightarrow f_D K^-) = z'_1 + z'_2. \quad (32)$$

The magnitude A_B is measured from $Br(B^- \rightarrow D^0 K^-)$, $A_{B'} r_B$ from $Br(B^- \rightarrow \bar{D}^0 K^-)$, while $|A_f|$ and $|\bar{A}_f|$ are obtained from measurements of the branching ratio and CP asymmetry of the $D \rightarrow f$ decays. This fixes the magnitudes $|z_{1,2}|$ and $|z'_{1,2}|$. Similarly $|z|$ and $|z'|$ are determined by measurements of $Br(B^\pm \rightarrow f_D K^\pm)$. If there is no direct CPV in charm decays then $|z_1| = |z'_1|$ and $|z_2| = |z'_2|$. This is what is assumed in the original GLW method, with the triangle constructions shown in Fig. 1. The magnitudes $|z_{1,2}|$, $|z'_{1,2}|$, $|z|$ and $|z'|$ are used to construct the two triangles (with two possible orientations), while the difference of the two angles, $\theta_1 = \arg(z_2/z_1)$ and $\theta_2 = \arg(z'_2/z'_1)$, determines γ ,

$$2\gamma = \theta_2 - \theta_1 = \arg(z_2 z'_1 / z_1 z'_2). \quad (33)$$

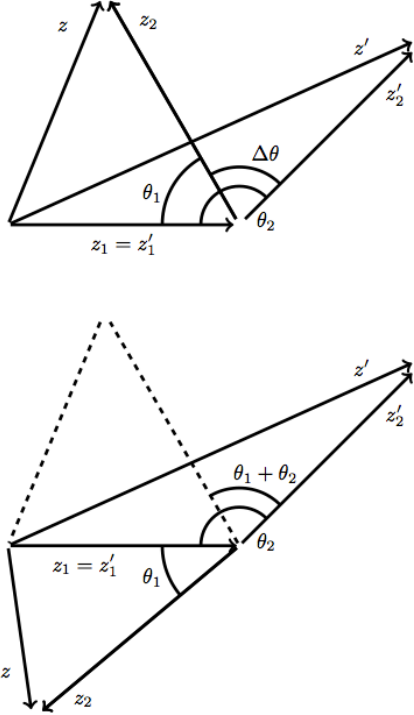


FIG. 1. The orientation of the triangles in GLW method with CP violation in SCS D decays included.

Algebraically, the difference of the two angles is given by

$$2 \sin^2 \left(\frac{\theta_2 - \theta_1}{2} \right) = 1 - c_1 c_2 \pm \sqrt{(1 - c_1^2)(1 - c_2^2)}, \quad (34)$$

where we shortened $c_1 \equiv \cos \theta_1$ and $c_2 \equiv \cos \theta_2$. The cosines are given directly in terms of the $|z_i|$ from the two triangle relations (31) and (32). For instance $c_1 = (|z_1|^2 + |z_2|^2 - |z|^2)/|2z_1 z_2|$, and similarly $c_2 = (|z_1'|^2 + |z_2'|^2 - |z'|^2)/|2z_1' z_2'|$.

In the derivation of (33) the assumption of no CPV in D decays entered at two steps

- *construction of the triangles.* The angle $\theta_1 = \arg(z_1/z_2)$ will no longer equal $\delta_B + \gamma$, but it will also contain the weak phase from charm decay amplitudes (and similarly for θ_2).
- *overlapping of the triangles.* In general $|z_1| \neq |z_1'|$ so that the two bases of the triangles in Fig. 1 no longer coincide.

If there is CPV in charm decays, so that $A_f \neq \bar{A}_f$, then the relation (33) gets modified to

$$\begin{aligned} \theta_2 - \theta_1 &= \arg \left(\frac{z_2 z_1'}{z_1 z_2'} \right) \Big|_f = 2(\gamma + \alpha_f) \simeq \\ &\simeq 2(\gamma - a_f^{\text{dir}} \cot \delta_f), \end{aligned} \quad (35)$$

where in the last equality we have only kept terms up to $\mathcal{O}(r_f)$. The difference of the two angles, $\theta_2 - \theta_1$ is thus now

related to γ through a final state dependent phase shift α_f (23). This corrects for the first of the two points above. It also shows explicitly that there is a shift symmetry $\gamma \rightarrow \gamma + \phi$, $\alpha_f \rightarrow \alpha_f - \phi$ in the problem. So unless this shift symmetry is broken γ cannot be determined.

The remaining question is how θ_1 and θ_2 are determined. If they are determined from the original GLW construction using triangles, then there is going to be an error in the extracted value of γ . Let's denote the phases determined in this way as θ_1^{dir} and θ_2^{dir} . They are given explicitly by $c_1^{\text{dir}} = (|z_1|^2 + |z_2|^2 - |z|^2)/|2z_1 z_2|$ and similarly for c_2^{dir} with $z_i \rightarrow z_i'$. The relation between the correctly and incorrectly determined phases is obtained after some algebra to be

$$\sin^2 \left(\frac{\Delta \theta^{\text{dir}}}{2} \right) = \sin^2 \left(\frac{\Delta \theta}{2} \right) + \mathcal{C}(\theta_1, \theta_2) \times a_f^{\text{dir}}, \quad (36)$$

where we again kept only terms up to $\mathcal{O}(r_f)$, and for shortness of notation defined $\Delta \theta^{\text{dir}} \equiv \theta_2^{\text{dir}} - \theta_1^{\text{dir}}$ and $\Delta \theta \equiv \theta_2 - \theta_1$. The multiplicative factor in the second term is

$$\mathcal{C}(\theta_1, \theta_2) = \frac{1}{4} \left(\frac{|z_1|}{|z_2|} - \frac{|z_2|}{|z_1|} \right) \tilde{\mathcal{C}}(\theta_1, \theta_2) \simeq \frac{1}{4r_B} \tilde{\mathcal{C}}(\theta_1, \theta_2), \quad (37)$$

with

$$\tilde{\mathcal{C}}(\theta_1, \theta_2) \equiv (c_1 - c_2) \left[1 + \sqrt{(1 - c_1^2)(1 - c_2^2)/(c_1 c_2)} \right]. \quad (38)$$

To the order we are working this function can be evaluated using c_i^{dir} instead of c_i .

Using GLW without taking into account CPV in charm one would conclude that $\Delta \theta^{\text{dir}}$ equals 2γ , a conclusion that is incorrect up to the shifts shown in (35) and (36). To stress this let us define

$$\gamma^{\text{dir}} \equiv \Delta \theta^{\text{dir}}/2. \quad (39)$$

The relation between the incorrectly determined and the correct values for γ are then from (35) and (36) given by

$$\gamma \simeq \gamma^{\text{dir}} + a_f^{\text{dir}} \left[\cot \delta_f - \frac{1}{4r_B \sin(2\gamma)} \tilde{\mathcal{C}}(\theta_1, \theta_2) \right], \quad (40)$$

or numerically, using $r_B = 0.1$, $\gamma = 68^\circ$,

$$\gamma = \gamma^{\text{dir}} + a_f^{\text{dir}} \left[\cot \delta_f - 3.6 \tilde{\mathcal{C}}(\theta_1, \theta_2) \right]. \quad (41)$$

We see, that the effect vanishes in the limit of no direct CP violation, $a_f^{\text{dir}} = 0$. Since the sensitivity to γ is proportional to r_B , part of the shift $\gamma - \gamma^{\text{dir}}$ is relatively enhanced by $1/r_B$. This results in a shift that is a factor of a few times the value of a_f^{dir} for $B \rightarrow DK$ decays. So until the precision on γ does not reach a level of a few percent this shift could even be ignored. This is not true for γ extraction from $B \rightarrow D\pi$. There the ratio of the two amplitudes is in this case

$$r_B(D\pi) \sim \left| \frac{V_{us} V_{cd}}{V_{cs} V_{ud}} \right| r_B \sim \lambda^2 r_B \sim \mathcal{O}(0.5\%). \quad (42)$$

Therefore the corrections due to CP violation in charm on the γ extraction in the case of $B \rightarrow D\pi$ decays is $\mathcal{O}(1)$, since it is enhanced by an extra factor of $1/\lambda^2 \approx 25$, and needs to be taken into account.

IV. CONCLUSIONS

We have provided a strategy that can be used to correct the extraction of γ from $B \rightarrow DK$ and $B \rightarrow D\pi$ decays for the effects of direct CP violation in charm decays. The extraction of γ requires that there is no CP violation in Cabibbo allowed and Cabibbo suppressed D decays, while the observed CP violation in singly Cabibbo allowed D decays can be included using our formulas. For this the knowledge of direct CP asymmetry in $D \rightarrow f$ decays, a_f^{CP} , is needed, along with the knowledge of the relative strong phases δ_f between the interfering amplitudes in the D decays. This strong phase can be obtained from high precision charm factory running at $\Psi(3770)$ in conjunction with CP violation measurements in D Dalitz plot analyses, e.g. in $D \rightarrow K_S \pi^+ \pi^-$. Alternatively, it can in principle be determined also in $B \rightarrow D^{(*)} K^{(*)}$ decays as well, if at least two different types of B decays with several different $D \rightarrow f$ decays are considered. Another approach, valid to first order in the ratio of penguin and tree matrix elements in $D \rightarrow f$ SCS decays, r_f , is also possible [22]. The shifts due to $a_{CP}^{\text{dir}} \neq 0$ in $BR(B \rightarrow f_D K)$ and $BR(B \rightarrow f_D \pi)$ are the same at $\mathcal{O}(r_f)$. The γ extraction can thus be appropriately modified to this order to include $a_{CP}^{\text{dir}} \neq 0$, if $B \rightarrow f_D K$ and $B \rightarrow f_D \pi$ decays are used simultaneously.

If CP violation in D decays is ignored in the extraction of γ , the resulting shift is of the order of $\mathcal{O}(a_f^{\text{dir}}/r_B) \sim \text{few degrees}$. With increasing precision of γ determination the inclusion of CPV in D decays will therefore soon become important. For the future γ extraction from $B \rightarrow D\pi$ decays the shift in γ is $\mathcal{O}(1)$ so that the inclusion of the CP violating effects will be essential from the start.

Note added: While this paper was being finalized Ref. [24] appeared, which stresses the importance of including a_f^{dir} .

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Appendix A: Complete expressions

In this appendix we collect the complete expressions for CP asymmetries and CP averaged branching ratios including direct CP violation in charm. The CP averaged branching ratios are

$$\begin{aligned} \text{Br}(B \rightarrow f_D K) &\equiv \\ &\equiv \frac{1}{2} (|A(B^- \rightarrow f_D K^-)|^2 + |A(B^+ \rightarrow f_D K^+)|^2) = \\ &= A_B^2 (A_f^T)^2 \left[1 + r_B^2 + r_f^2 + r_f^2 r_B^2 + \right. \\ &\quad + 2r_B \cos \delta_B \cos(\gamma + 2\phi_f^T) + 2r_f \cos \delta_f \cos \phi_f \\ &\quad + 4r_f r_B \cos \delta_b \cos \delta_f \cos(\gamma + \phi_f + 2\phi_f^T) \\ &\quad + 2r_f r_B^2 \cos \delta_f \cos \phi_f \\ &\quad \left. + 2r_f^2 r_B \cos \delta_B \cos(\gamma + 2\phi_f + 2\phi_f^T) \right], \end{aligned} \quad (\text{A1})$$

while the CP asymmetry is

$$\begin{aligned} A_{CP}(B \rightarrow f_D K) &= \frac{A_B^2 (A_f^T)^2}{\text{Br}(B \rightarrow f_D K)} \times \left[2r_B \sin \delta_B \sin(\gamma + 2\phi_f^T) + 2r_f \sin \delta_f \sin \phi_f + 4r_f r_B \sin \delta_b \cos \delta_f \sin(\gamma + \phi_f + 2\phi_f^T) \right. \\ &\quad \left. + 2r_f r_B^2 \sin \delta_f \sin \phi_f + 2r_f^2 r_B \sin \delta_B \sin(\gamma + 2\phi_f + 2\phi_f^T) \right], \end{aligned} \quad (\text{A2})$$

where $\text{Br}(B \rightarrow f_D K)$ is given in (A1).

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- [1] R. Aaij *et al.* [LHCb Collaboration], Phys. Rev. Lett. **108** (2012) 111602 [arXiv:1112.0938 [hep-ex]].
 - [2] CDF collaboration, Phys. Rev. Lett. **109** (2012) 111801 [arXiv:1207.2158 [hep-ex]].
 - [3] Heavy Flavor Averaging Group, winter 2011, available at <http://www.slac.stanford.edu/xorg/hfag>
 - [4] Y. Grossman, A. L. Kagan, Y. Nir, Phys. Rev. D **75**, 036008 (2007) [hep-ph/0609178].
 - [5] J. Charles *et al.* [CKMfitter Group Collaboration], Eur. Phys. J. C **41** (2005) 1 [hep-ph/0406184]; Winter 2012 update.

-
- [6] M. Gronau and D. London, Phys. Lett. B **253** (1991) 483.
 - [7] M. Gronau and D. Wyler, Phys. Lett. B **265** (1991) 172.
 - [8] D. Atwood, I. Dunietz and A. Soni, Phys. Rev. Lett. **78** (1997) 3257 [hep-ph/9612433].
 - [9] A. Giri, Y. Grossman, A. Soffer and J. Zupan, Phys. Rev. D **68** (2003) 054018 [hep-ph/0303187].
 - [10] J. Zupan, arXiv:1101.0134 [hep-ph].
 - [11] J. Zupan, Nucl. Phys. Proc. Suppl. **170** (2007) 65.
 - [12] C. C. Meca, J. P. Silva, Phys. Rev. Lett. **81** 1377-1380 [hep-ph/9807320].

- [13] A. Amorim, M. G. Santos, J. P. Silva, Phys. Rev. D **59** (1999) 056001 [hep-ph/9807364].
- [14] J. P. Silva, A. Soffer, Phys. Rev. D. **61** (2000) 112001 [hep-ph/9912242].
- [15] Y. Grossman, A. Soffer and J. Zupan, Phys. Rev. D **72** (2005) 031501 [hep-ph/0505270].
- [16] J. Brod, A. L. Kagan and J. Zupan, Phys. Rev. D **86** (2012) 014023 [arXiv:1111.5000 [hep-ph]].
- [17] J. Brod, Y. Grossman, A. L. Kagan and J. Zupan, JHEP **1210** (2012) 161 [arXiv:1203.6659 [hep-ph]].
- [18] D. Pirtskhalava and P. Uttayarat, Phys. Lett. B **712** (2012) 81 [arXiv:1112.5451 [hep-ph]].
- [19] B. Bhattacharya, M. Gronau and J. L. Rosner, arXiv:1207.0761 [hep-ph].
- [20] E. Franco, S. Mishima and L. Silvestrini, JHEP **1205** (2012) 140 [arXiv:1203.3131 [hep-ph]].
- [21] G. Isidori, J. F. Kamenik, Z. Ligeti and G. Perez, Phys. Lett. B **711** (2012) 46 [arXiv:1111.4987 [hep-ph]]; Y. Hochberg and Y. Nir, Phys. Rev. Lett. **108** (2012) 261601 [arXiv:1112.5268 [hep-ph]]; G. F. Giudice, G. Isidori and P. Paradisi, JHEP **1204** (2012) 060 [arXiv:1201.6204 [hep-ph]]; W. Altmannshofer, R. Primulando, C. -T. Yu and F. Yu, JHEP **1204** (2012) 049 [arXiv:1202.2866 [hep-ph]]; T. Feldmann, S. Nandi and A. Soni, JHEP **1206** (2012) 007 [arXiv:1202.3795 [hep-ph]]; G. Hiller, Y. Hochberg and Y. Nir, Phys. Rev. D **85** (2012) 116008 [arXiv:1204.1046 [hep-ph]]; B. Keren-Zur, P. Lodone, M. Nardecchia, D. Papadopulo, R. Rattazzi and L. Vecchi, Nucl. Phys. B **867** (2013) 394 [arXiv:1205.5803 [hep-ph]]; C. -H. Chen, C. -Q. Geng and W. Wang, arXiv:1206.5158 [hep-ph]; C. Delaunay, J. F. Kamenik, G. Perez and L. Randall, arXiv:1207.0474 [hep-ph]; L. Da Rold, C. Delaunay, C. Grojean and G. Perez, arXiv:1208.1499 [hep-ph].
- [22] The LHCb Collaboration, LHCb-CONF 2012-032.
- [23] G. C. Branco, L. Lavoura and J. P. Silva, *CP Violation*, Oxford University Press (1999).
- [24] W. Wang (2012) [hep-ph/1211.4539].